ARES

Introduction to Photogrammetry

Using perspective analysis to infer the trajectory of incoming munitions

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About the author

Bjørn Holst Jespersen

Bjørn Holst Jespersen is a Danish architect and designer, with extensive experience in producing architectural illustrations. Following the uprisings in the Middle East and North Africa (MENA) region in 2011, he began applying his skills to solving conflict image analysis problems and has since worked extensively with open source intelligence. He has worked with Armament Research Services (ARES) on several projects. In addition to his perspective analysis and reverse projection work, Mr Jespersen has also assessed image integrity and validity and conducted other advanced image analysis tasks. He is currently taking a sabbatical to focus on creative pursuits.

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AVOID the area

RECORD all relevant information

MARK the area to warn others

SEEK assistance from the relevant authorities

Disclaimer

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Cover image: *The remnants of a 300 mm artillery rocket fired from the 9K58* Smerch *multiple-barrel rocket launcher in Kramatorsk, Ukraine, on 10 February 2015 (Source: Vos Iz Neias).*

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Introduction

This ARES Field Guide introduces the use of photogrammetry—defined as the science of reading or obtaining measurements from a photograph—to investigate ordnance and its impacts.

The guide does not provide a comprehensive catalogue of photogrammetric techniques, but demonstrates how a particular technique, perspective analysis, can be applied to a photograph, and how it can be used to advance research in conflict zones. After reading, the reader should not expect to be able to perform advanced perspective analysis, but he or she should have gained a valuable understanding of the technique's possibilities. The basic principles of perspective analysis, as presented here, should be understandable to anyone with a basic knowledge of geometry; while some trigonometry is used to make calculations, an understanding of how trigonometry works is not essential to a meaningful reading.

The guide looks specifically at one case study, showing how to infer the incoming trajectory (and, potentially, the range[1]) of a projectile from a photograph of a building damaged by a rocket in Donetsk, Ukraine. In this particular case, extracting this information would help narrow down a search for the launch position of the rocket, which might in turn point to who perpetrated the attack.

It is important to note that the type of analysis presented in this report is an example of what could be done with the material available and does not represent an ideal investigation. For registering and documentation purposes, a single photograph is not optimal. More photographs of the room in question could supply extra information and a better understanding of the situation and might also enable or simplify the use of certain software. Whilst multiple images are often available to the analyst, this study focuses on a 'minimum standard' needed to perform perspective analysis with reasonable confidence.

It is also worth noting that a close inspection of the photograph in this case study reveals a slight curvature to the longer, straighter contours close to the edges of the frame. This is due to camera distortion, a phenomenon that is not addressed in detail in this case. The distortion in this case is of such a small order that it was possible to compensate for it while performing the analysis, avoiding a significant impact on the result. In other cases, camera distortion might have to be handled differently.

This Field Guide focuses on the techniques used in this particular case, rather than the information gathered in subsequent investigations; additional information was later collected to support the analysis herein.

[1] This is not explored in depth in this study but relies on information such as impact angle.

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Background to the Case Study

Figure 1.1 *The original photograph as obtained by ARES in 2014 (source: Harriet Salem/ARES).*

The photograph shown in Figure 1.1. provides the primary source material for this analysis. The image was provided to ARES by a journalist working in Ukraine, in 2014. The imagery in the photograph was supplemented by additional information, which includes metadata, witness interviews, and contextual information. The metadata from the photo indicates it was taken at 10:21 a.m. on 15 October 2014. The geographic coordinates, 48°01'42.34"N, 37°47'40.49"E, were also included. This information is consistent with the information provided by the photographer, and other contextual information gathered from those present and other sources.

Eyewitness accounts indicate that the damage was caused by the impact of a cargo rocket section, which created the hole in the floor, and, at least, the hole in the ceiling nearest to the viewpoint (indicated by the yellow tape measure in Figure 1.1).

> *In the photographs of the roof (...) one impact hole can be seen. Eyewitnesses report that a "long tube" was stuck in this hole and worked its way down through the building. Rebels later removed the tube.*
>
> *In the photographs of the loft (...) there are two impact holes. Eyewitnesses could not be specific about how the second hole was made (there did not seem to be two entry holes in the roof) but speculated it was created by a submunition explosion.*
>
> - Confidential source

Figure 1.2 is annotated to show one possible interpretation involving another projectile, which could explain the second hole in the ceiling and the distorted shape of the hole in the floor. Whether caused by a submunition, rogue fragment, or a secondary projectile or rocket, the second hole in the ceiling is not examined further in this study. The munition in question was most likely a cargo rocket, based upon an ARES assessment of other available evidence, and this is assumed to be the case in this guide. Contextual images were also provided to ARES, however these are not assessed herein.

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Figure 1.2 *An annotated version of the original photograph, showing the potential path of a second projectile or rocket impact (source: Harriet Salem/ARES).*

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Perspective analysis

As explained above, the nearest[2] hole in the ceiling and the hole in the floor are assumed to have been caused by a cargo rocket. Under that premise, the photograph potentially contains information pointing towards the launch site of this rocket, which would be along a line drawn in the horizontal direction between the centre of the hole in the floor and the centre of the nearest hole in the ceiling.[3] It is of interest to extract this information, as it could help determine which party to the conflict perpetrated the attack. However, it is clearly not a case of just using rulers overlaid on the image to estimate a direction. Instead, in order to take measurements from a photograph (with a view to determining directions), it is necessary to account for distortions caused by perspective. Perspective analysis is a way to handle this.

1. Key concept: vanishing points

A simple way to describe perspective distortion would be to say that identical objects appear smaller when they are further away from the viewpoint. This also means that an object's dimensions appear smaller when viewed along the line of sight rather than across it.[4] The latter effect is known as foreshortening.

The rules for understanding this distortion are well established; they date to the Renaissance and are frequently used in the fields of visual arts and architecture to construct realistic views of imagined places, or to visualise designs. The concept of 'vanishing points' is central to this understanding because these create reference points for the rest of the photograph.

The term 'vanishing points' is used to describe a phenomenon in which, when viewed through the eye or camera, parallel lines appear to converge at a distant point.[5] One consequence is that different groups of parallel lines—running horizontally and not viewed at a 90 degree angle—will have their vanishing points on a shared horizontal line: the horizon (see Figure 1.3). It also means that if the line of sight is not completely level, vertical lines will also appear to converge. In the photograph used in this study, the camera was pointed below the horizon, which means that both the horizontal and vertical lines have a vanishing point.[6]

[2] The whole which appears nearest from the perspective of one viewing the photograph.

[3] Using the centre of the holes suggests these are assumed to be the centre of the primary impact. However, while the circular shapes of the holes in the ceiling makes this a reasonable assumption, the shape of the hole in the floor raises questions. This will be addressed later in the study.

[4] If you take, for instance, a pencil and hold it in front of you horizontally and look straight at its side, its length will occupy a certain percentage of your field of view. If you then turn the pencil, say, 60 degrees around a vertical axis, the percentage of your field of view occupied by the side of the pencil diminishes, or the pencil could appear to be shorter – hence the term 'foreshortening'.

[5] The exception to this rule is parallel lines viewed from a 90-degree angle, which will remain parallel.

[6] In a case where the line of sight is completely vertical (that is, the camera was pointed straight up or straight down), all horizontal lines will be viewed at a 90-degree angle, and will therefore not have vanishing points.

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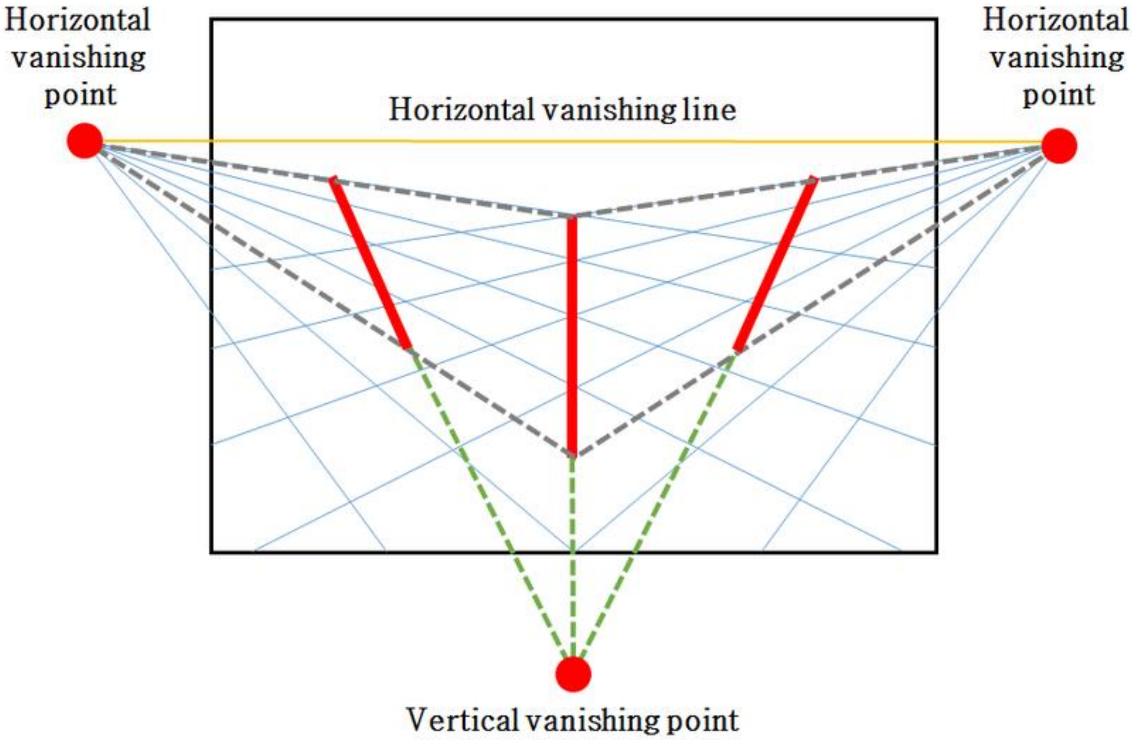

Figure 1.3 *Visualisation of vanishing points and the horizontal vanishing line (horizon) (source: Jaehook, Paik & Yoon, 2016)*[7].

2. **Locating vanishing points for reference**

The first stage of the analysis process is to locate vanishing points for reference. They are located by extending at least two lines assumed to be parallel in reality; the place where they intersect is their vanishing point. This process is repeated to locate vanishing points for all directions needed to perform the next steps in the analysis (see Figure 2.1 below). Placing these lines can be quite difficult. The process largely depends on identifying the clearest directional contours. A simple way to accomplish the placing of these lines is to load the photo into a vector graphics programme and annotate it with different sets of coloured lines (Figure 2.1).

When vanishing points for at least two different horizontal directions have been located, the horizon can be established by drawing a line through these. If vanishing points for more horizontal directions have been located, these should all be on the horizon. In practice, these will often not be exactly in line, but with the right subsequent adjustments the result can end up being sufficiently reliable. If the camera is tilted, the horizon in the image will be tilted too. If so, it can be helpful to manually rotate the image to align the horizon with the horizontal plane.

[7] Jaehoon, Jung, Joonki Pak & Inhye Yoon. 2016. 'Object Occlusion Detection Using Automatic Camera Calibration for a Wide-Area Video Surveillance System'. *Sensors*. 16(7): 982.

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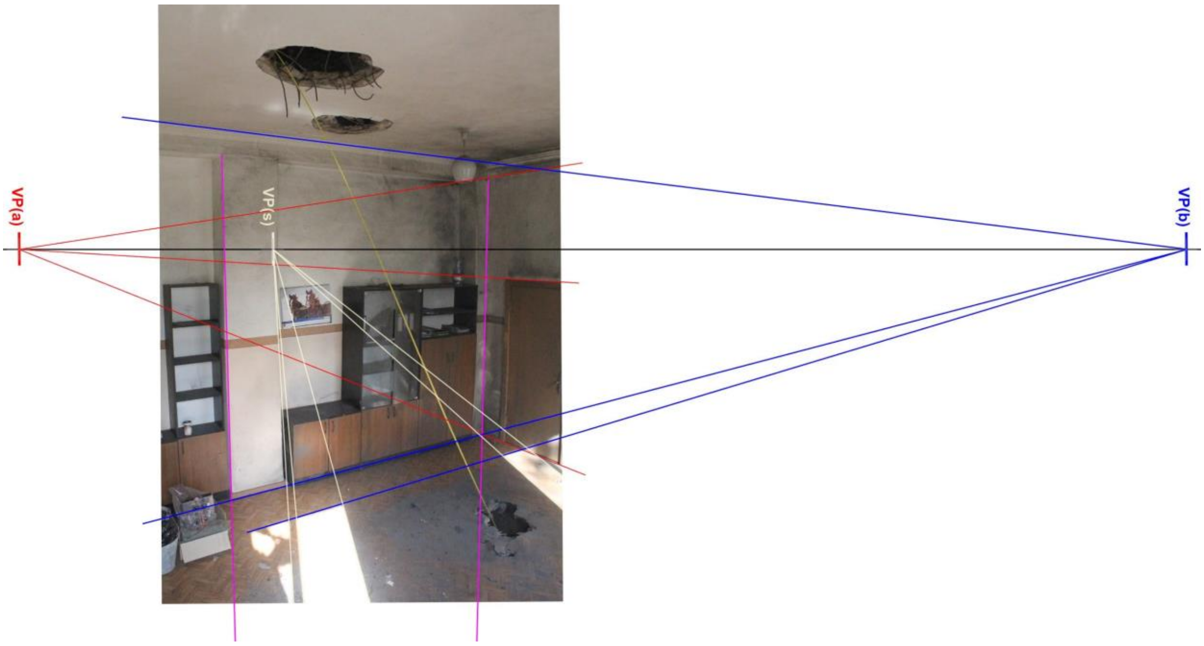

Figure 2.1 *The original photograph overlaid with lines to identify horizontal vanishing points (red and blue lines) and establish the horizon (horizontal black line) (source: Harriet Salem/ARES).*

Figure 2.1 shows the vanishing points (VP) for the primary directions in the photograph. *VP(a)* is the vanishing point for the horizontal direction of the wall to the right of the photograph (vanishing to the left) and *VP(b)* is for the horizontal direction of the wall to the left (vanishing to the right). *VP(a)* and *VP(b)* are located using the edge of the floor and ceiling compared against their respective walls (marked with blue lines for the left-hand wall and red for the right).

Identifying vanishing points for additional horizontal directions will, as noted earlier, generally help to make the analysis more accurate. In this case, an additional direction that could be assessed was the direction of the visible sunlight, designated *VP(s)*. By using the relevant edges of the visible sunlight on the floor (marked with white lines) it is possible to add an extra point which can be used to test and adjust the horizon. Additionally, because the far edge of the sunlight cast on the floor must be assumed to be parallel with the facade wall of the room, extending that line to see that it also runs to the vanishing point of the horizontal direction of the left wall tells us that the facade wall and the left-hand wall in the photograph are parallel. This corroborates the assumption of an orthogonal layout of the room, and, on that basis, also helps to locate the precise vanishing point of the direction marked with blue lines. Similarly, the line extending from the top of the door helps to locate the vanishing point of the direction marked with red lines.[8] The vanishing point for the vertical direction (pink lines) is too far down to be shown in the illustration, but this set of lines does in fact have a vanishing point which has been located in a similar way as the three other identified vanishing points, and which is needed to perform the analysis. It will hereafter be referred to as *VP(v)*.

[8] In practice, it is often best to extend as many lines as possible for each respective group. They probably won't all intersect at the same point, but from there they can be adjusted until a satisfactory result is achieved.

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The process of locating these points rests on assumptions about how images (the photograph) represent reality (the room). In this example, it is assumed, first, that the floor and ceiling are level and that the walls are vertical. Small inaccuracies with respect to angles are inevitable in the construction of buildings, but it is reasonable to expect that a twentieth century building with an orthogonal layout in a major city of the former Soviet Union conforms to, or is very close to, the assumption that floors and ceiling are level, walls are vertical and the angles between walls are 90 degrees. The second assumption is that transposing the coloured lines 'by eye' produces sufficient accuracy. Based on simulations, in this case the risks to accuracy are estimated to be of such an order that it still makes sense to apply this type of analysis.

3. **Working with points**

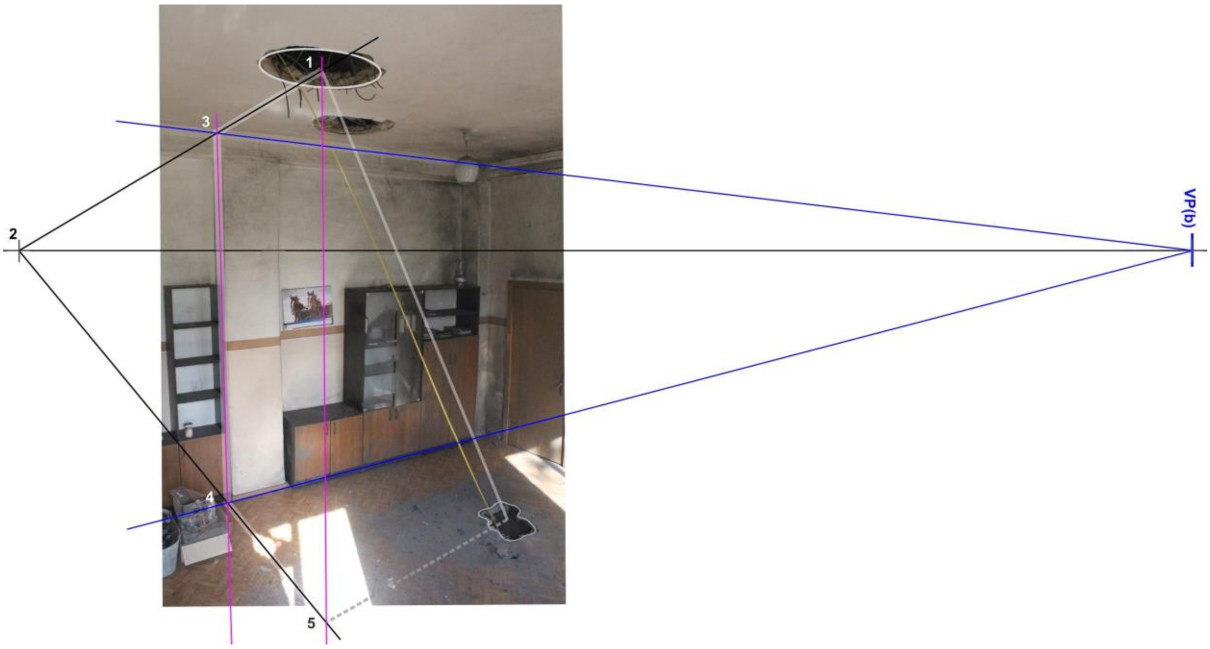

Figure 2.2 *The original photograph overlaid with an illustration of the process of locating an imagined point (source: Harriet Salem/ARES).*

As noted above, the aim is to establish the horizontal direction between the hole in the floor and the nearest hole in the ceiling. A way to do this is to locate the point on the floor directly beneath the hole in the ceiling, and then draw a line between this and the centre of the hole in the floor. So, with the vanishing points located and the horizon marked, the next step is to locate that point.

Figure 2.2 shows the completed process and is used for reference during the following steps. The desired point, marked on the floor as *5*, is located at the intersection of two lines (pink and black), which is a typical situation when performing perspective analysis.

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Those two lines can be produced, and *point 5* identified, via the following series of steps:

1. A line that must run through *5* is the line running straight down from, or rather, vertically through, *point 1*. Since the vanishing point for the vertical direction, *VP(v)*, has been located, this line can be drawn by simply connecting *VP(v)* to *point 1* (the rightmost pink line in Figure 2.2).

2. To produce the intersecting line—which determines where the pink line drawn during step 1 meets floor level—a horizontal line running through *point 1* is mirrored to floor level. The horizontal line at ceiling level is drawn as a line from the centre of the nearest hole in the ceiling (*point 1*) to a random point on the horizon (*point 2*)[9]. *Point 2* now marks the vanishing point for the direction represented by *line 1–2*.

3. Any line sharing this new vanishing point (*point 2*) will be parallel to *line 1–2*; what is required here is the one directly under *line 1–2* at floor level. *Point 2* can be used as the starting point to construct this new line, but one more point directly under *line 1–2* at floor level is still necessary. Here the left wall—since it is assumed to be vertical—can be used. The wall being vertical means that, by connecting *point 3* to *VP(v)*, a line is drawn which follows the wall vertically, and therefor intersects with the bottom edge of the wall at a given point (*point* 4). This is directly under *point 3* and at floor level, as required.

4. The final step to locate *point 5* is to draw a line from *point 2* through *point 4* and extend it until it intersects with the *line 1–VP(v)* created during step 1.

To sum up: since this new line (shown in Figure 2.2 in black) shares its vanishing point with *line 1–2*, the two lines are parallel. And since the new line passes through a point at floor level (*4*) which lays directly under *line 1–2*, the line from 2 through 4 (*line 2–4*) also runs directly under *line 1–2* at floor level. It will therefore intersect with the vertical (pink) *line 1–VP(v)* directly under *point 1* at floor level which is where *point 5* is located.

After locating *point 5*, it is possible to mark the horizontal component of the munition's trajectory by drawing a line from *point 5* to the centre of the point of impact on the floor (shown with a dashed grey line in Figure 1.2).[10] Even if the angle between the marked trajectory and the left wall is still viewed in perspective—depending on the situation and one's confidence—it might be possible to

[9] Any direction from *point 1* could be used, but some will be more practical than others. The primary concern is to be able to read the intersections precisely, which means that the line at ceiling level should not meet the horizon at too sharp an angle, while still producing an intersecting line which does not intersect the line drawn in step 1 too sharply either.

[10] In this example the centre of impact is assumed to be the estimated centre of the hole in the floor. However, the irregular shape of this makes it unclear where the actual centre is. This will be discussed below, when margins of error are considered.

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make an estimate of the angle, which, combined with location information, would be sufficient to derive a useful estimate for the assumed bearing toward the launch position.

It is, however, important to be aware of the limits that this process imposes on confidence. For example, in this case, the assumption is that the rocket was not significantly deflected from its initial trajectory when passing through the building's structure. The confidence impact of a similar assumption will vary based on a number of factors, such as munition type, size, and weight, and the construction of the structure in question.

4. **Estimating Dimensions**

The process has now reached a point where it may be possible to estimate a bearing towards the assumed launch position 'by eye'. Depending on the situation, this could be a sufficient pointer for a continued search by other means (for example, a search of satellite imagery or, where possible, a search on the ground or by air). However, it is often possible to further refine the estimate. This section describes how to use vanishing points to compare a known dimension to an unknown which, in this case, is used to calculate the angle between the horizontal component of the trajectory and the left wall. The angle sought is illustrated in Figure 2.3a, where it is marked *a(x)*.

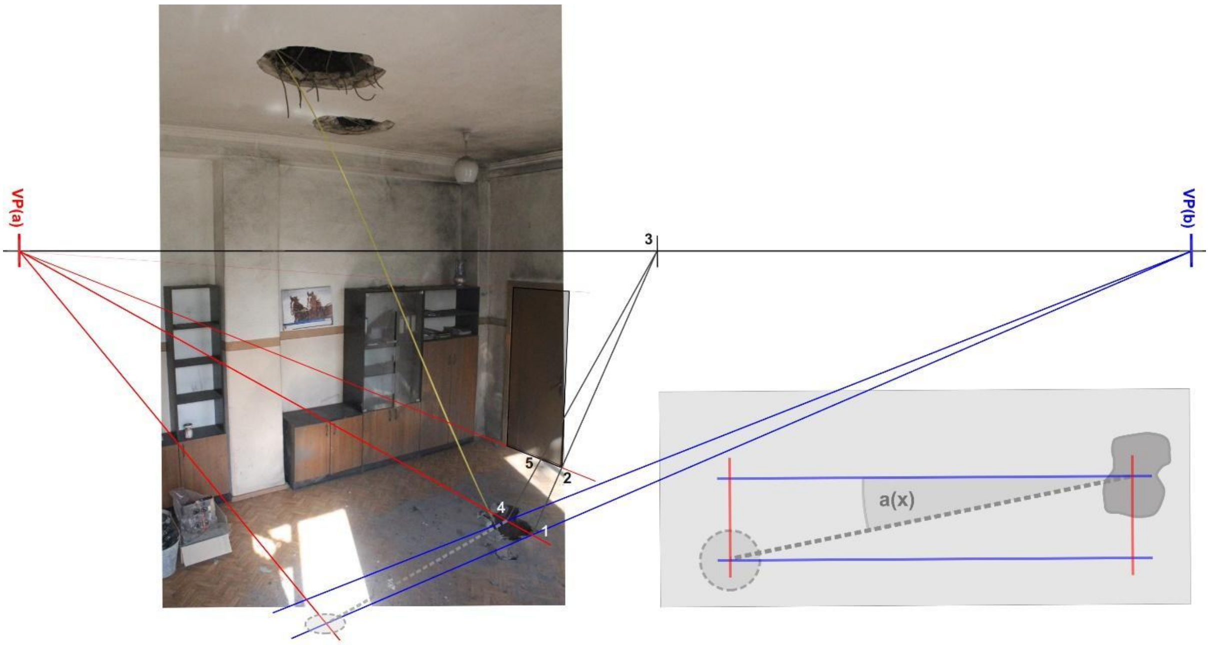

Figure 2.3 *The original photograph overlaid with an illustration showing the transferral of the length of the short leg for comparison (source: Harriet Salem/ARES).*
Figure 2.3a ***(inset, right)*** *a principal diagram of the situation on the floor (source: Bjørn Holst Jespersen/ARES).*

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This process uses trigonometry to calculate *a(x)*. For that it is necessary to create a right-angled triangle with the line representing the horizontal component as one of its sides. In this case—because the directions of the right and left walls create a right angle—this can be achieved by extending lines to both of the vanishing points for the two walls. These are drawn from both the estimated centre of the hole in the floor and from *point 5* in Figure 2.2, previously located on the floor. The line representing the horizontal component of the trajectory (marked by the grey dashed line in Figure 2.3) now forms the hypotenuse of two notional, identical right-angled triangles drawn on the floor, whose other, shorter, sides (legs) are parallel to the left and right walls. This means the dimensions of these legs can be estimated by comparing them to the size of the door and cupboards, which, because they are parallel to the walls, are also parallel to the legs. This is essential for comparison.[11]

The width of the door is used to estimate the length of the leg parallel to the right wall (*line 1–4*, in red in Figure 2.3), using the following steps:

1. Draw a line from the nearest end of the leg (*point 1*) to the nearest side of the bottom edge of the door (*point 2*). Extend it to the horizon (*point 3*). *Point 3* now locates the vanishing point for the direction from *point 1* to *point 2*, which makes it possible to draw *line 4–5* parallel to *line 1–2* from the other end of the triangle leg.

2. Draw a line from the furthest end of the leg (*point 4*) to the vanishing point for the direction of *line 1–2* (*point 3*). Because they share a vanishing point, *line 1–3* and *line 4–3* are parallel. *Point 5,* identified as the intersection of *line 4–3* with the bottom edge of the door, is therefore the same distance from *point 2* as *point 4* is from *point 1*. This means that the length of the leg (*line 1–4*) can now be transferred the bottom edge of the door (*line 2–5*).

If the two objects had been closer in size, this might have been sufficient to make an estimate of the transferred length based on the width of the door, but as they were not in this case, the initial estimate should be further refined.

[11] Unfortunately, no object whose size is known precisely is present in this image. However, for the sake of demonstration, this exercise will use a hypothetical figure (see text) for the door and cupboard sections.

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This can be done using the following steps:

1. Extend a line from *VP(v)*, the vanishing point for the vertical lines, upward through *point 5* to mark the transferred length of the leg (*line 1–4*) on the door (area shaded in Figure 2.4 and 2.4a).

2. Draw diagonal lines to locate the middle of the door and use lines from the relevant vanishing points (*VP(a)* and *VP(v)*) to subdivide width and height. Continue this process until a sufficiently precise estimate can be made (see Figure 2.4a).

Figure 2.4 *The original photograph overlaid with an illustration showing the early stages of measuring through a process of subdivision (source: Harriet Salem/ARES).*
Figure 2.4a ***(inset, right)*** *A closer look at the wall section containing the door after completion of the subdivision process (source: Bjørn Holst Jespersen/ARES).*

Measuring the transferred length indicates that the length of the shortest side of the triangle is 0.35 times the width of the door. Assuming the door is 75–85 cm wide[12], this corresponds to a leg length of 26.3–29.8 cm.

[12] Standard architectural references can provide guidance here, as can country-specific manufacturer brochures, building surveys, and other material.

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Figure 2.5 *The original photograph overlaid with an illustration showing the transferral of the long leg for comparison (source: Harriet Salem/ARES).*

The width of the cupboard is used to estimate the length of the triangle side parallel to the left wall (*line 1–4*, marked in blue in Figure 2.5):

1. Draw a line from the nearest end of the leg (*point 1*) to the nearest cupboard corner (*point 2*). Extend it to the horizon (*point 3*).

2. Draw a line from the vanishing point (*point 3*) to the furthest end of the leg (*point 4*). This transfers the length of this leg to the bottom edge of the cupboard (*line 2–5*).

As the transferred length matches more than two cupboard sections, a more precise estimate can be generated by subdividing the third section in perspective, in a similar way to before (see Figure 2.6a).

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Figure 2.6 *The original photograph overlaid with an illustration showing the early stages of measuring through a process of subdivision (source: Harriet Salem/ARES).*
Figure 2.6a ***(inset, right)*** *A closer look at the area with the relevant cupboard section after completion of the subdivision process (source: Bjørn Holst Jespersen/ARES).*

Measuring the transferred length indicates that the triangle side is 2.4 times the width of a cupboard section. Assuming the cupboard section is 80–100 cm wide, this corresponds to a length of 192–240 cm.

The dimensions of these two notional identical right-angled triangles can now be used to calculate the direction of the horizontal component of the trajectory. Combining the largest figure for the shortest side (29.8 cm) with the smallest figure for the other side (192 cm) produces the estimated triangle with the widest *a(x)* angle. Combining the smallest figure for the shortest side (26.3 cm) with the largest figure for the other side (240 cm) produces the estimated triangle with the narrowest *a(x)* angle.

These angles can be calculated using trigonometry: inverse tangent of (29.8/192) = 8.8 degrees and inverse tangent of (26.3/240) = 6.3 degrees. If the door and cupboard sizes are inside the intervals used, then the value for angle *a(x)* falls within this narrow interval. However, because there are a number of possible sources of error, it is appropriate to expand the interval. One source of error is the estimated sizes of the elements used for comparison. If these have been estimated wrongly, the resultant calculations will likely be wrong too, and it will be impossible to predict by how much.[13]

[13] In practice, if there is doubt about the estimated ranges for the sizes of elements used for comparison, it is best to change those estimated ranges. In this instance, since the estimates were made with confidence, they will be left as they are.

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Another source of uncertainty is the assumption about the location of the centre of impact in the floor. The irregular shape of the hole, in contrast with the hole in the ceiling, makes it less obvious where the centre of impact is. It seems equally possible that the primary centre of impact is in a position where it will either halve the angle a(x) or add about a half to it. Subtracting half from the lowest figure gives 3.1 (6.3 minus 3.2) and adding half to the highest gives 13.2 (8.8 plus 4.4). This leads to an expansion of the interval, resulting in an interval from 3.1 to 13.2 degrees. To that must be added the possibility of error due to inaccuracy in locating the vanishing points. The effect of this possibility has been simulated in relation to the reverse perspective projection technique described below. The results of the simulation lead to an estimate of +/- 2 degrees contribution to the total error margin. Another +/- 0.5 degree is estimated to cover possible inaccuracies in the building. If these numbers are used here too, the interval is expanded further to an interval from 0.6 to 15.7 degrees. In other words—since it can be seen directly that the size of the angle *a(x)* is more than 0—in this case applying the comparison technique will only help put a cap on the higher end of the estimated size of the angle.

If this seems disappointingly imprecise, it should be remembered that 7.6 degrees (3.2 plus 4.4) of the estimate's interval originate from the uncertainty about the hole in the floor, with the uncertainty about the exact sizes of the elements used for comparison adding another 2.5 degrees. This means that the lack of clarity about the situation accounts for 10.1 degrees while the comparison technique itself in this estimate accounts for just 5 degrees.

It is also important to note that in less complex situations where, for example, just one vertical element of unknown height can be compared to another vertical element of known height by using a vanishing point, this technique is a lot easier to apply and can deliver answers with minimal error margins.

As in previous stages of the process, it is important to be aware of limits on confidence. This process assumes that the walls in the room photographed meet at a 90-degree angle (a reasonable assumption for the reasons listed above). Furthermore, the viability of this method depends on the presence of objects in parallel to one another and other reference points. In the example, where, due to the lack of elements of known size, this technique was primarily applied for demonstration purposes, the process hinged on the fact that the sides of the triangle were parallel to the front of the door and the cupboards respectively, and on the assumption it was possible to give a sufficiently narrow estimate of their sizes.

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5. **Working with geospatial data**

So far, the analysis has delivered a direction interval relative to a room. To put this to use in a further search for the (assumed) launch site, the position of the room has to be located and its orientation has to be determined. The following section therefore describes how to combine the results of the analysis with supplemental information to generate an absolute position and a bearing towards the launch position.

First, the position. Coordinates for the position of the photographer when taking the photograph were provided. When entering these in Google Earth or other GIS imaging software, the position shown is within a building complex.

Second, the orientation of the room. The building complex has an orthogonal layout with facades facing in only four directions. By combining this with the pattern of sunlight visible in the photograph at a known local time (10:21 a.m.), all rooms except those facing southeast can be eliminated.

Figure 2.7 *Overhead imagery from Google Earth, with GPS photo markers, indicators of the location of impact sites, and a directional estimate (source: Google Earth/ARES).*

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Pinpointing the room exactly within the building complex should in this case—with coordinates provided—be relatively easy. However, when working through the historical satellite imagery in Google Earth it can be seen that the imagery shifts around a little under the map marker pins. Given the (assumed) length of the distance to the launch site, pinpointing the room in the imagery is not strictly necessary to find the trajectory, but it can help avoid errors in a photogrammetric model by checking data against the real world. Figure 2.7 indicates a possible location of the room within the building complex based on the coordinates provided, combined with suspected damage visible from an analysis Google Earth's historical imagery.[14]

The final step is to measure the bearing of the room and determine the direction interval. In the photograph, the left wall is parallel to the building's face, and, as mentioned above, the direction of the sunlight reveals that this must be one of the southeast faces. The direction of these can be measured using Google Earth or another GIS tool, which indicates a bearing of 229.8 degrees. Subtracting the angles of the interval, described in the previous section, produces an interval for the assumed bearing towards the source between 214.1 and 229.2 degrees.

[14] The suggested location of the room is a small distance away from the marked position of the photograph. As long as any error is a difference of a similar size, this should not have a significant effect on the result.

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Reverse Perspective Projection

Perspective analysis in this case produced a direction interval rather than a specific bearing because of a lack of clarity about the situation and the size of the reference objects used for comparison. Had confidence in the estimates been higher, this process would have produced a narrower direction interval.

However, in some cases it will be possible to work further without using estimated sizes of objects, or even in the absence of reference objects of known sizes. It is sometimes possible to produce a direction interval by applying another technique: reverse perspective projection. This technique can produce a plan drawing of the room, from which the direction can be measured more directly. Perspective projection produces a perspective view from scale drawings (like plan drawings). Reversing this process produces a plan drawing from a perspective view (like the photograph under study in this guide). In cases such as this one, where both techniques are viable, the uncertainty derived from using reference objects of an unknown size can be reduced or eliminated.

Understanding how this technique works may require experience with perspective projection. This is not a common skill, and it is beyond the scope of this report to bring readers to that level. The following section will therefore limit itself to a demonstration of how this technique may be applied. Instead of providing detailed step-by-step explanations, it will only give a brief description of the process and its principles.

1. Key concept: picture plane

The 'picture plane' is an imaginary surface located between the viewpoint (in this example, the camera) and the object being viewed (the room), oriented perpendicularly to the line of sight. By first transferring points to this plane, features from the perspective view can be transferred to a plan drawing.

When producing a perspective view, the picture plane is perpendicular to the desired line of sight from the viewpoint. It can be placed in any position in front of the viewpoint, but the further away, the larger in size the perspective. When this process is reversed, the picture plane is drawn parallel to the horizon at any practical distance underneath the perspective view. In this case, it can be helpful to imagine the photograph 'standing' on the picture plane (see Figure 3.1).

2. Locating the viewpoint

When producing a plan drawing, positioning the viewpoint relative to the picture plane is critical because points are transferred using their line(s) of sight from the picture plane to the viewpoint.

In this case, because the photograph is uncropped, the latitudinal position is determined by the centre of the photograph. The longitudinal position can be located because the left and right walls are assumed to meet at a 90-degree angle. This means that the lines of sight to the two vanishing points transferred to the picture plane extend from the viewpoint at a 90-degree angle as well

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(Figure 3.1). In accordance with a geometrical rule[15], this means that a semicircle arcing between the two transferred vanishing points intersects with the vertical centre line in this point (shown with dashed lines, Figure 3.1).

3. **Transferring points**

After placing the picture plane and viewpoint, relevant features can be transferred to the plan drawing.

First, transfer floor-level points from the perspective image by extending vertical lines until they intersect with the picture plane (e.g. *line 1–2*).

Second, transfer these points from the picture plane to the plan drawing. In Figure 3.1, the transfer began by fixing the inner corner of the room directly on the picture plane (*point 2*), but the plan could have been drawn at any distance in front of the viewpoint by placing the corner along the relevant line of sight (line from *viewpoint* to *point 2*).[16]

Then, the direction of the two walls can be drawn relative to the inner corner (*point 2*). These are parallel to the lines of sight extending from the viewpoint to the transferred vanishing points on the picture plane (marked with red and blue lines in Figure 3.1).

[15] The rule is Thale's Theorem, which deals with semi-circles and is a special case within the general 'inscribed angle theorem' which deal with full circles.
[16] The plan will not be in any known scale, but in this case that doesn't matter since it is an angle we are looking for, and those remain unaffected. However, it is also important to note that if just one object of known horizontal dimensions can be placed confidently in the plan drawing, the scale of the plan can be calculated.

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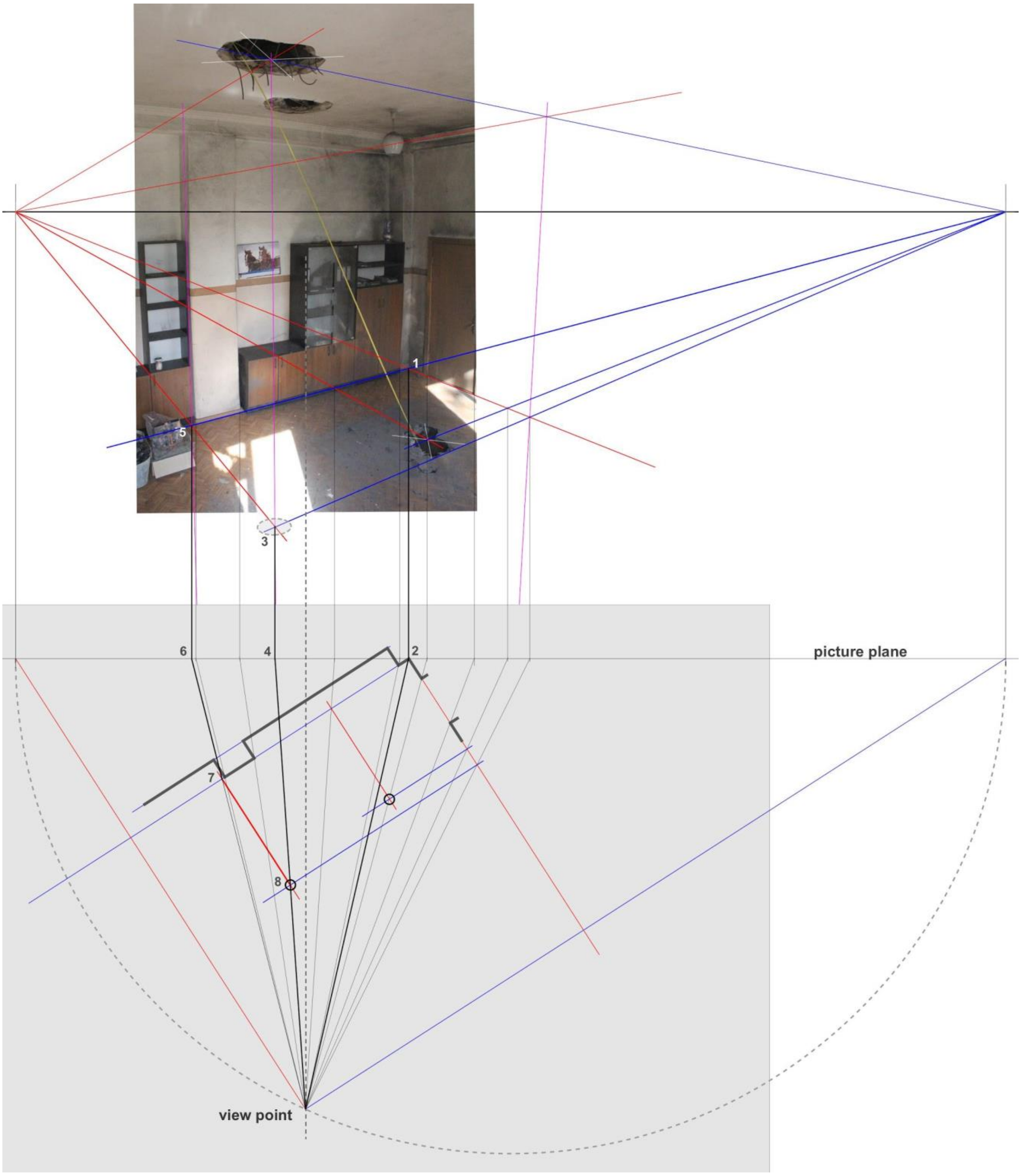

Figure 3.1 *The original photograph with the reverse perspective projection process performed (source: Harriet Salem/ARES; Bjørn Holst Jespersen/ARES).*

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4. **Transferring more points**

From here the analysis could continue point transferral in different directions. Highlighting one possible path, this section looks at how to transfer the point on the floor directly under the hole in the ceiling (*point 3*) to the plan drawing (*point 8*).

First, transfer the point straight down to the picture plane (*point 4*) and mark the line of sight (*line 4–viewpoint*). The point will be placed on this line. Then, transfer *point 3* to the left wall in the perspective image using the right wall direction (*point 5*), and then straight down to the picture plane (*point 6*). To locate *point 3*'s position in the plan drawing, extend a line from *point 6* to the viewpoint, intersecting the left wall (*point 7*).[17] Finally, draw a line from *point 7* parallel to the right wall. Since this line will also pass through point 3's position in the plan, its intersection with the *line 4–viewpoint* marks the right location on the plan for the point under the hole in the ceiling (labelled *point 8* in the plan drawing).

5. **Calculating direction from the plan drawing**

With the plan drawing complete, the trajectory can be measured relative to the left wall. Figure 3.1 indicates the result is 8.2 degrees. This corresponds to a bearing of 221.6 degrees for the direction of the horizontal component. This figure, however, has to be given a measure of uncertainty assessed to be about +/- 6.6 degrees (the reasons for which are noted above, and will be looked at in more detail in Outcomes and Uncertainties, below). Therefore, the result here is a direction interval of bearings from 215 to 228.2 degrees.

As mentioned earlier, the reverse perspective projection technique can be used to eliminate the uncertainty arising from a comparison to objects of an unknown size. The confidence increase in this case would have been 2.5 degrees, however, since the estimated uncertainty arising from the distorted shape of the impact hole in the floor (+/- 50% of *a(x)*) is relative to the size of the angle *a(x)* and since the result of the reverse perspective projection for *a(x)* is towards the higher end of the interval found by comparing sizes, this error margin differs in absolute numbers and is 0.6 degrees larger in the reverse perspective projection example. Accordingly, the confidence increase is of just 1.9 degrees. The direction interval which covered a span of 15.1 degrees, is now assessed at 13.2 degrees.

[17] In this example it was easier to identify a line (marked in blue) parallel to the left wall, instead of the left wall itself. This parallel line is therefore used.

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6. **Extracting more information from reverse projection and plan drawings**

Further detail can be added to the reverse projection by adding the sunlight pattern and the second hole in the ceiling to the plan drawing.

Adding the second hole can help determine its origin. One theory is that it was caused by fragmentation resulting from the rocket breaking up upon impact, or part of the structure that had broken off and was pushed downward. A view from the centre of the hole in the floor through the holes in the ceiling might have revealed whether any of the ceiling holes, in combination with the hole in the floor, were aligned with holes further up in the building structure, which would have added important evidence to help understand the situation more clearly.

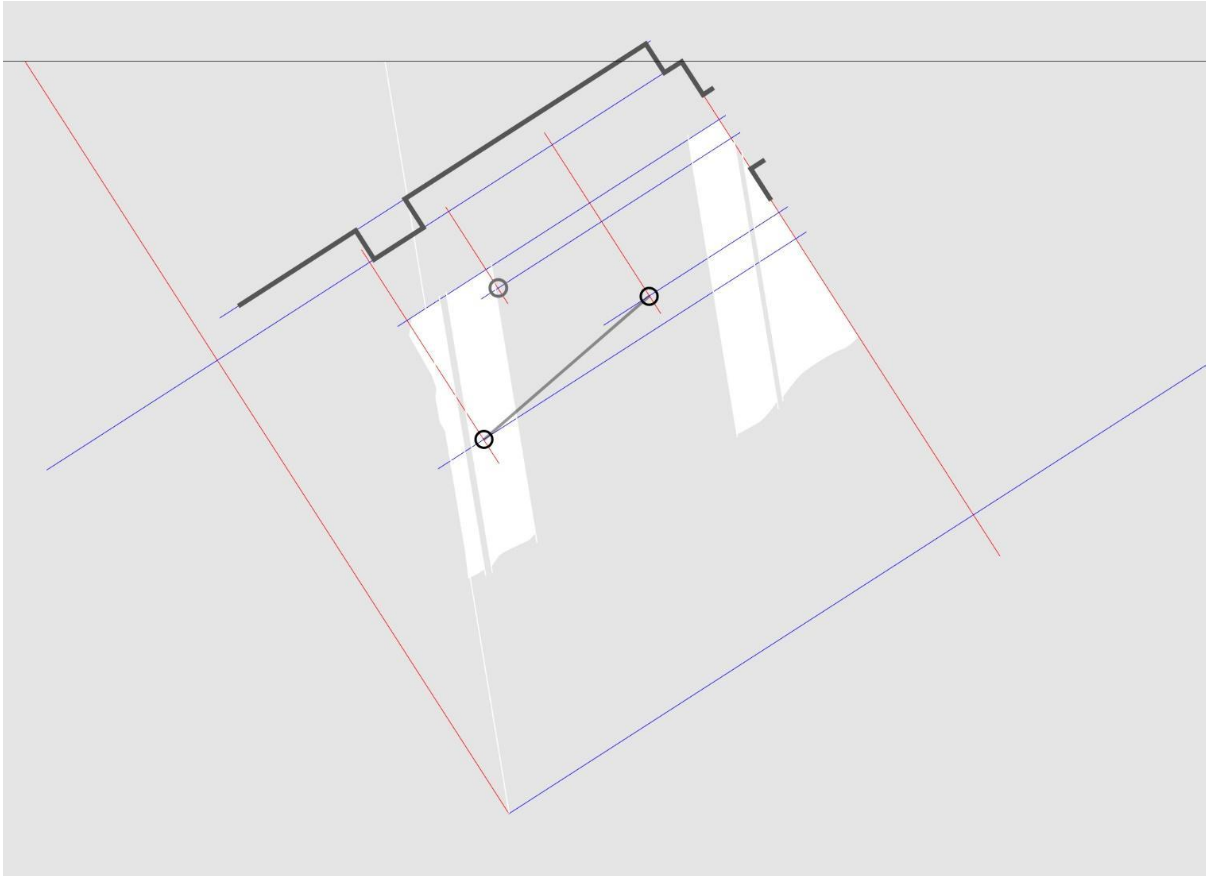

Figure 3.2 *Plan drawing derived from reverse perspective projection with more details added (source: Bjørn Holst Jespersen/ARES).*

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The sunlight pattern can be used to cross-reference other data. If the time and date are firmly established, but the location is less certain or the orientation—as in this case—unknown, the direction of the sunlight can function as a compass. If time was the unknown factor, it could be determined by measuring the solar azimuth. In this example, the bearing is 163.2 degrees (based on the known location). Online calculators indicate this angle corresponds to a time of 11:17 a.m., assuming a GMT+2 time zone with daylight saving time (DST).

This differs significantly from the time included in the photograph's metadata (10:21 a.m.), which undermines confidence in—at least—the reasoning behind the establishment of the orientation of the room. However, since the difference is close to one hour, it is possible that the photographer either had the camera set to a different time zone or that the clock in the camera had not been adjusted for DST. Since the original analysis, the photographer has provided additional data to ARES researchers that corroborates this theory.

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Outcomes and Uncertainties

Although perspective analysis can shed light on ordnance and its impacts based on photographic evidence, it is not always possible to confidently apply the techniques covered here. Like any model-dependent process, uncertainty over how closely the real world resembles the simplified model creates sources of error, as does a requirement to estimate object dimensions without a scale. When there is insufficient confidence in the accuracy of underlying assumptions, these techniques may require further contextual or corroborative evidence to reach a conclusion.

In this example, the process first assumes that the supplied metadata was correct and that the photographer took the photo at the claimed time and place. Second, the process' overall precision varies according to operator skill in locating vanishing points and placing lines. This varies according to a range of factors including image resolution and the capabilities of different graphics programs. Third, the physical world is never identical to the models used. For example, the process assumes that the walls in the photo meet at a 90-degree angle, the floor and ceiling are level, and the walls are vertical. These factors and assumptions, in a case like this, only make it possible to produce a basis for an estimated figure because a measure of inaccuracy is inevitable.

Simulations were developed and conducted to test the effects of uncertainty in locating vanishing points and of inaccuracies in the building. These led to an estimate of a +/- 2.0-degree error margin, as a result of the risk of inaccuracy when locating the vanishing points and imprecision when moving points around, constructing rectangles, and so on. An additional +/- 0.5 degrees is—in view of the assumptions made about the room—estimated to cover possible inaccuracies in the building.

These figures were also used when comparing sizes. Uncertainty about the exact placement of points, due to the irregular shape of the hole in the floor, makes an additional margin of error necessary. As described, the shape of the hole can be interpreted so that the centre of the primary point of impact is not in the middle of the hole, but instead closer to either end of the 'longish' hole where it will either roughly halve the measured angle a(x) or increase it by about half.

As a result, this is estimated to add +/- 4.1 degrees to the uncertainty for the reverse perspective projection. Together these factors add up to a +/- 6.6 degrees error margin. The direction interval estimate resulting from this is therefore 13.2 degrees wide, which does not include estimates of possible effects on the course of the assumed rocket before passing through the room. This might not be as narrow an interval as could be hoped for, but given the 8.2 degrees originating from the uncertainty about how to interpret the hole in the floor, 8.2 degrees is a minimum margin of error, even if everything else could be measured with absolute accuracy.

The process also assumes that the horizontal component of the direction between the two holes is indicative of the direction towards the launch site. Even if eyewitness reports are correct and the assumptions of a cargo rocket they (in combination with other observations) led to are correct, it is not possible to know what affected the projectile's course before it passed through the room under study. The structural components of the building and meteorological conditions are two variables

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that may further affect the accuracy of photogrammetric techniques. For instance, the rocket may have struck heavier parts of the building structure or had its flight path affected by wind conditions that could affect conclusions drawn from the photograph and supporting information.

The unexplained second hole in the ceiling highlights these factors of uncertainty and shows the need to couch a narrow technical method, like perspective analysis, in a broader investigative context. Its primary utility is as a step in a process of narrowing down a search—of satellite imagery, in this case—rather than as a standalone method to draw a definitive conclusion.

With these caveats in mind, this report estimates that in this case there would be reasonable cause to start the search for a launch site between bearings of approximately 213 degrees and 230 degrees. However, since meteorological data or elevation data have not been factored in, their effects and other contextual effects on the assumed cargo rocket's flight path should be considered, and the estimated bearings adjusted accordingly, before laying out the definitive search area.

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